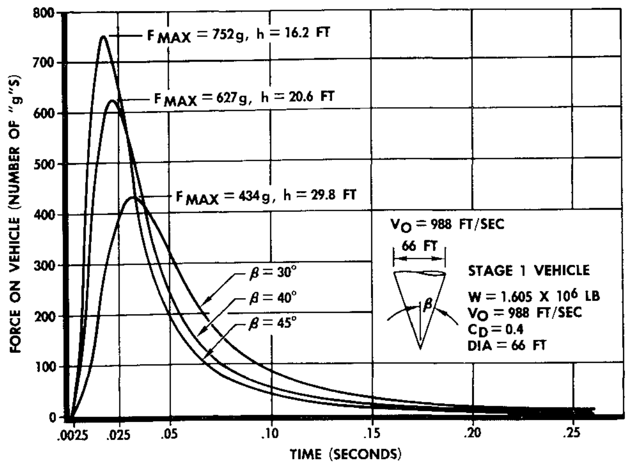

Fig. 1 Hydrodynamic force on vehicle as a function of time.

velocity potential arising from an ellipsoid circumscribed about the cone and then making wetting and free-surface corrections.

The right-hand side of Eq. (2) can be interpreted thusly. The time rate of change of momentum

$$\frac{d}{dt}\left(m\frac{dh}{dt}\right) = \frac{d}{dt}\left[(M_0 + M)\frac{dh}{dt}\right] = (M_0 + M)\frac{d^2h}{dt^2} + \frac{dM}{dt}\frac{dh}{dt} \quad (4)$$

However

$$\frac{dM}{dt} = \frac{dM}{dh}\cdot\frac{dh}{dt}$$

and knowing M as defined by Eq. (3), we get

$$\frac{dM}{dt} = 3k\frac{\rho_W}{g}h^2\tan^3\beta\frac{dh}{dt} \quad (5)$$

Upon substituting Eqs. (3) and (5) into (4) and hence into (2), and noting that the instantaneous cross-sectional area of a vehicle with a conical nose is $A = \pi h^2 \tan^2\beta$, the result is the following equation of motion:

$$A - Bh^3 - Ch^2\left(\frac{dh}{dt}\right)^2 = \frac{d^2h}{dt^2}(D + Eh^3) \quad (6)$$

where $A = W$, $B = \rho_W\pi\tan^2\beta$, $C' = \rho_W C_D\pi\tan^2\beta/2g$, $D = W/g$, $E = k(\rho_W/g)\tan^3\beta$, and $C = C' + 3E$. This is a second-order nonlinear differential equation with constant coefficients, the initial conditions being that at $t = 0$, $h = 0$ and $dh/dt = V_0$ = entry velocity.

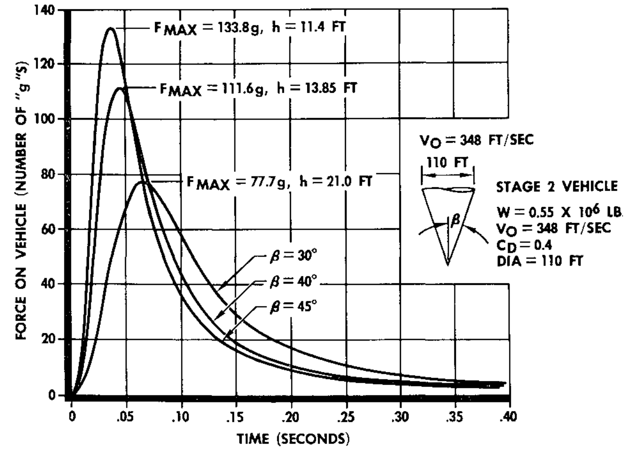

Fig. 2 Hydrodynamic force on vehicle as a function of time.

Equation (6) has been programmed for solution on the IBM 7090 computer using a digital integration program titled "Didas" (for Digital Differential Analyzer). Reference 3 includes the derivation of Eq. (6), its solution by the forementioned program, solution by a Taylor Series approximation, and calculations and graphs of force-time histories for various configurations of recoverable boosters.

Figures 1 and 2 describe the force-time history obtained using the "Didas" solution for two configurations which were studied at Douglas. Noted on the figures are the depths of penetration at which the maximum forces occur.

A comparison of the maximum force determined from this force-time analysis for the stage 2 vehicle of Fig. 2 with values predicted from other analyses is shown in Table 1.

The results in column 4 were calculated from values of the dimensionless "virtual" mass of the fluid k which Watanabe[4] calculated from his experiments. Shiffman and Spencer's values were calculated using Eq. 2.3.10 of Ref. 2.

Column 5, denoted by A, represents the values which are obtained if the impact effect is neglected as in Ref. 1. This can be done by setting $E = 0$ and $C = C'$ in Eq. (6) of this note.

Column 6, denoted by B, represents the value of the force that is calculated from an analysis which makes the assumption, namely, that the virtual mass is picked up by the vehicle with the same instantaneous speed of the vehicle. This assumption is contrary to the one inherent in the calculation of the forces shown in columns 2 and 3 which is that the virtual mass is picked up by the impacting vehicle from a state of rest.

In conclusion it should be emphasized (as was mentioned by the authors of Ref. 1) that unless the geometric configuration is such that it provides a low ballistic parameter, the water entry velocity will be so great as to produce a g level that would be prohibitive even for the most rugged structures.

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[1] Davidheiser, J. A. and Armstrong, J. M., "Recovery of large rocket boosters by a free-fall water-impact method," IAS Preprint 62-53 (January 1962).

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Landing Loads for Ocean-Recovered Rocket Boosters: Reply by Author to G. Stoffmacher

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In his note, "Hydrodynamic impact of conical-nosed vehicles during vertical water entry,"[1] Stoffmacher claims that the authors of Ref. 2 have neglected the virtual mass of liquid in-

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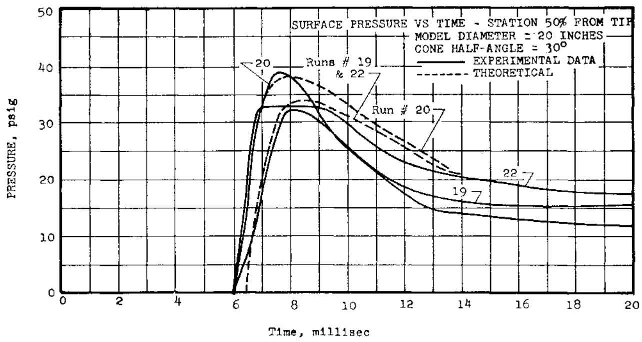

Fig. 1 Surface pressure vs time (station 50% from tip); impact velocity = 115 fps.

volved in the water impact phenomenon and therefore have underestimated the forces involved on the vehicle during water entry. On pp. 14–15 of Ref. 2, however, there is a discussion of the impact forces which specifically includes the effect of virtual mass. In fact, Eq. (8), which is an expression for the maximum axial force during impact, shows clearly the inclusion of the dimensionless virtual mass k. The equation of motion to which Stoffmacher refers is found on p. 19 of Ref. 2. As stated on p. 19, this equation is for the *subsurface* trajectory *only* and is used to estimate the maximum depth of water penetration.

While the virtual mass effect should be included in the subsurface equation, the result of neglecting this effect is to yield conservative estimates of the maximum depth, which was acceptable for determining design features of the recoverable booster.

Using Eq. (8) of Ref. 2, the maximum water entry deceleration for the examples Stoffmacher presented were calculated. Table 1 presents a comparison of the deceleration for the respective equations.

As can be seen, the forces predicted by Eq. (8), Ref. 2 are similar to Stoffmacher's results. The differences result since Eq. (8) is theoretically valid only when the mass of the vehicle is much greater than the mass of water displaced. For the cases considered here the mass of vehicle is greater than mass of water displaced by approximately a factor of 4 for stage 1 and a factor of 4.5 for stage 2. Thus the "low density" effect of these bodies accounts for the difference in deceleration. For design purposes these values, plus the knowledge that they will be low due to the forementioned effect, are adequate.

It would seem that the author of Ref. 1 had perhaps overlooked Eq. (8) in Ref. 2 in his review of the paper. The authors had attempted to show as clearly as possible that the virtual mass concept was included in the consideration of water entry phenomenon. Reference 3 was a study which considered solution of the equation of motion, again including virtual mass, for deceleration as a function of time. Figures 17 through 22 of Ref. 3 illustrate the results of this analysis.†

Table 1 Maximum impact deceleration

Semi-vertex cone angle, deg	Stoffmacher (Ref. 1, Figs. 1 & 2)		Ref. 2 (Eq. 8, p. 15)	
	Stage 1	Stage 2	Stage 1	Stage 2
30	434	77.7	358	65
40	627	111.6	582	100
45	752	138.8	694	124

Of considerable importance for determining structural behavior is the ability to predict the local pressure distribution over the cone as a function of time during impact. Figures 20 and 21 of Ref. 3 show the pressure-time history on a large booster during water entry.

In order to verify the theoretical load predictions, scale models of a large booster were constructed. The models were $\frac{1}{30}$ scale and 20 in. in diameter and approximately 65 in. in length. Froude scaling of the impact velocities was assumed and the models were instrumented with photocon pressure transducers. The experimental data for impact velocities in the order of 100 fps indicated good correlation with the theoretical and test results. Figure 1 shows the results for the transducer located at a station 50% of the distance from the cone apex.

References

[1] Stoffmacher, G., "Hydrodynamic impact of conical-nosed vehicles during vertical water entry," J. Spacecraft **2**, 222–223 (1964).

[2] Davidheiser, J. A. and Armstrong, J. M., "Recovery of large rocket boosters by a free-fall water-impact method," IAS Preprint 62–53 (January 1962).

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† Reference 4 also shows the results of this study.

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